# Metacognitive Judgments in Learning the Derivative Concept Using the Virtual Lab Strategy

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## Abstract

This study examined a classroom intervention through a virtual laboratory as a didactic strategy that was based on the understanding of the derivative concept applied to the problem of the tangent line. The Pretest-Posttest methodology was used to test if there are significant differences in the learning of the derivative concept, compared to the traditional teaching approach. The results indicate that a statistical analysis implies three categories in relation to competencies developed by the students: solution of equations, identification of terms, and conceptual questions. There was a difference in the variances of populations for the competence of solving equations, and the experimental group showed better performance in; the other two categories showed similar performance in both groups. Thus, we consider the proposed method as an alternative form of teaching the derivative concept. The metacognitive regulation strategies implemented by the students during the didactic intervention process were also identified.

**Key Words:** Derivative concept, didactics of mathematics, comparative analysis cognition, metacognitive judgements.

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## Juicios metacognitivos en el aprendizaje del concepto de derivada utilizando la estrategia del laboratorio virtual


### Resumen

Esta investigación implementó una intervención en el aula, utilizando un laboratorio virtual como estrategia didáctica basada en la comprensión del concepto de derivada, aplicado al problema de la línea tangente. Se aplicó la metodología *Pretest-Posttest* para ver si existen diferencias significativas en el aprendizaje del concepto de derivada en comparación con el enfoque de enseñanza tradicional. Los resultados indican que al realizar el análisis estadístico se identifican tres categorías en relación a las competencias desarrolladas por los estudiantes: solución de ecuaciones, identificación de términos y cuestiones conceptuales. Se encontró diferencia en las varianzas de poblaciones para la competencia de resolución de ecuaciones, obteniendo mejor desempeño en el grupo experimental; las otras dos categorías muestran un desempeño similar en ambos grupos, por lo que consideramos el método propuesto como una forma alternativa de enseñar el concepto de derivada. Finalmente, se identificaron las estrategias de regulación metacognitiva aplicadas por los estudiantes durante el proceso de intervención didáctica.


**Palabras Clave:** Concepto de derivada, didáctica de las matemáticas, análisis comparativo, cognición, juicios metacognitivos.

### Introduction

Math learning problems are a reference point in the development of science and didactics of mathematics. Over the last years, researchers have been aware of the difficulties in learning and teaching mathematics from different theoretical perspectives and they have focused on the analysis of the overall structure of problem-solving processes (Davis et al., 2011; Kitcher,1984; Kline,1998; Lakatos, 2015; Polya, 2004; Cheung and Wong, 2011; Ho and Lowrie, 2014; Martin et al., 2017).

The lack of understanding of basic mathematical concepts is a common phenomenon that influences the learning of new concepts in mathematics (Llorens et al.,1996; Vinner and Dreyfus, 1989; Vinner, 2002). This work of research shows that problems in understanding concepts manifest in various ways with a common root: the existence of contradictory conceptual images.

For Vinner and Dreyfus (1989), the image that students associate with the concepts generates a contradiction between the formal concept and its own conception, in short, both, the one produced in the student's mind and the formal concept are not synchronously integrated into the construction of learning, or do not correspond to each other. As a result of this phenomenon, the student gives a purely mechanical interpretation of algorithmic-algebraic application to the concepts.

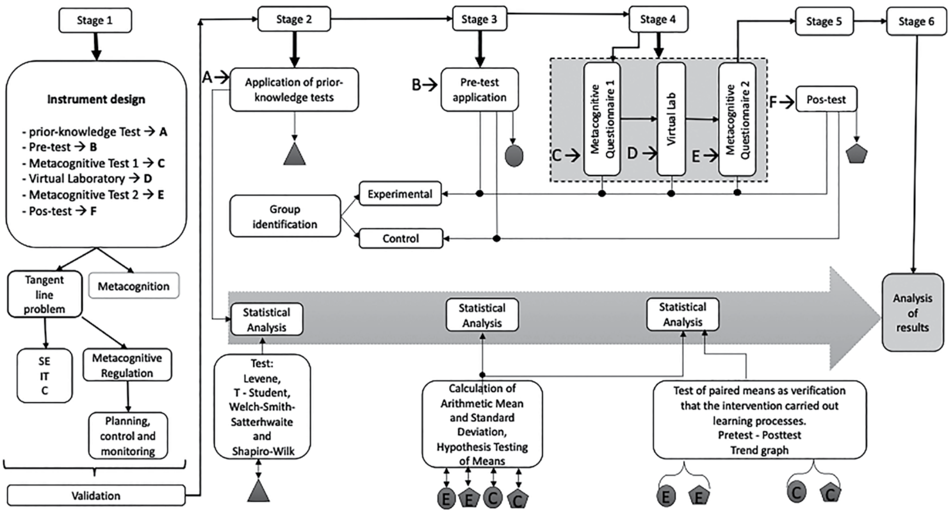


**Figure 1.** Methodological design
Source: Prepared by the authors

The purpose of this work of research is to evaluate the use of the virtual laboratory strategy and the problem of the tangent line, starting from the theory of metacognitive regulation, which according to several authors, allows improving the problem-solving skills (Flavell, 1970,1975,1979,1999, Hacker et al., 1998; Huitt, 1997; Mayer,1998; Özsoy and Ataman, 2009; García et al., 2016; Gok, 2010; Händel and Dresel, 2018; Izzati, 2021; Londoño et al., 2021; Sandi-Urena et al., 2012; Vaccaro and Fleming, 2018; Ye et al., 2019).

## Research Method

The first step was to establish two groups, experimental and control made up by students of two differential calculus courses of an engineering program at the same university. A previous knowledge test was administered to both groups and the exploratory analysis showed that there were no significant differences according to the t-test, therefore, the equality of variances was verified through the Shapiro-Wilk test. Accordingly, the control group and the experimental group are freely designated.

The second step consisted of using a Pretest-Posttest model in both groups. In the experimental group, the posttest was administered after performing the intervention with the virtual laboratory strategy, accompanied by a constant monitoring of the metacognitive regulation strategies associated with learning the concept of derivative. The analyses in this second instance were done through a media difference t-test between groups.

Figure 1 shows the phases of this quasi-experimental study (Pedhazur and Schmelkin, 1991), which has a comparative nature as a complement with a non-equivalent groups design (Campbell and Stanley, 1995).

### Phase 1: Design and validation of tests with metacognitive questionnaires

In this phase, five tests were designed: Prior knowledge test, Pretest, Metacognitive Questionnaire 1, Metacognitive Questionnaire 2, and Posttest (Table 1). In each test, problem statements and/or problematic situations were proposed to address issues related to the concept of the tangent line problem.



The problems are framed into categories based on the Pretest-Posttest model of Bringula, Basa, and De la Cruz and Rodrigo (2016), *i.e.*, identification of terms (IT), solution of equations (SE), and conceptual questions (C). Simultaneously, at least one metacognitive regulation question is addressed to include problem-solving aspects, as proposed by Buitrago and García, (2011).

The prior knowledge test was assessed through 11 conceptual questions divided into three categories, which were defined according to the types of knowledge proposed by Woolfolk (2010), declarative and procedural knowledge.

According to Buitrago and García (2011), to inquire about their evaluation process within metacognitive regulation, at the end of each conceptual question, the students are asked to answer the question: What difficulties do you have in solving the question?

**Table 1.** Test designed for this research with the topics addressed about the concept of the tangent line problem and metacognitive regulation processes in each phase

| Phase 2 | Phase 3 | | Phase 4 | | Phase 5 |
|---|---|---|---|---|---|
| Pre-Knowledge | Pretest | Metacognitive Questionnaire 1 | Virtual lab | Metacognitive Questionnaire 2 | Postest |
| Tangent and Secant line<br><br>Slope of a tangent line<br><br>Function.<br><br>Continuity of a function.<br><br>Limit of a function. | Analysis of the tangent line of a function at a point.<br><br>Estimation of the slope of a curve and the representation of the tangent line as a function.<br><br>Geometric interpretation of the derivative | Questions regarding:<br><br>Troubleshooting the virtual lab of the tangent line before solving the proposed problem | Graphical approach to the concept of derivative, using the tangent line. problem | Questions regarding:<br><br>Troubleshooting the virtual lab of the tangent line after solving the virtual lab proposed problem | Analysis of the tangent line of a function at a point.<br><br>Estimation of the slope of a curve and the representations of the line tangent as a function.<br><br>Geometric interpretation of the derivative |
| Metacognitive regulation processes at each stage | | | | | |
| Evaluation | Evaluation | Evaluation<br>Control<br>Monitoring | Doesn't apply | Evaluation<br>Control<br>Monitoring | Evaluation |

Source: Prepared by the authors



The Pretest and Posttest had 12 questions about the topics listed in Table 1. Each question is categorized and subcategorized as shown in Table 2. Table 3 presents the questions used in the Pretest and Posttest metacognitive questionnaires, with their respective associated regulatory process. Student score(s) was averaged according to equation (1) proposed in Bringula et al. (2016).

**Table 2.** Pretest and Posttest structure.

| Type of knowledge | Categories | | | Question number |
|---|---|---|---|---|
| | General** | Sub category | Sub-category definition | |
| Procedural | Equation Solution (SE) | Traits of the graph (RD) | Identifies fundamental features in the construction of the graph of the geometric interpretation of the derivative | 1 |
| | | Error Identification (IE) | It consists in pointing out within a procedure for solving a problem, the existence of an error in that process. | 2 |
| Declarative | Conceptual Questions (C) | Production of the equation of a tangent line (PT) | Analysis of the line tangent to a function at a point, slope estimation, and graphical representations of the line tangent to the curve $f(x)$ | 4 |
| | Term Identification (IT) | Next Step (PP) | It consists of observing an incomplete sequence of troubleshooting a problem and identifying the next step for its resolution. | 1 |
| | | Term Identification (IT) | It consists of recognizing two or more different algebraic expressions and relating them in a single one | 4 |

Note: *According to (Woolfolk, 2010); **Taken from Pretest-Posttest model (Bringula et al., 2016)
Source: Prepared by the authors

## 2.2. Phases 2, 3 and 4: Pretest, Intervention and Posttest



After establishing the control and experimental groups, the Pretest phase was carried out.  Table 4 shows the steps of such process.

**Table 3.** Classification of questionnaire items against Metacognitive regulation processes.

| Metacognitive regulation process* | Prospective Questionnaire Items | Retrospective Questionnaire Items |
|---|---|---|
| Planning | How do you think you can solve this virtual lab? | Do you think you managed to solve the virtual lab the way you expected? If not, what has changed your initial expectations? |
| Control or monitoring | Do you think it takes anything to solve the virtual lab? | What were the main difficulties you encountered in the virtual lab solution? |
| Evaluation | Where do you think the difficulty of the virtual lab lies? | Describe how efficient you think your strategy for solving the virtual lab was |

Note: *according to the proposal of Buitrago and García (2011)
Source: Prepared by the authors

**Table 4.** Steps in a row in Phase 3 and 5, Pretest and Posttest respectively

| Step | Description |
|---|---|
| 1 | Application and Test Rating in the control and experimental groups. |
| 2 | Literal transcription of all answers to metacognitive regulation questions. |
| 3 | Consolidation, classification and tabulation of the results against the questions grouped by:<br>3.1) Categories (IT, SE, C)<br>3.2) Subcategories for metacognitive regulation questions defined in step 3 of the presaber phase |

Source: Prepared by the authors

## Results and Analysis



This work of research was developed through questionnaires that included items related to planning, control, and evaluation as indicators of metacognitive regulation processes. Accordingly, a statistical analysis was done using the arithmetic mean, the standard deviation, the hypothesis test, the difference of means and the test of paired means.

### 3.1. Metacognitive Questionnaires

Table 3 shows the classification of items from the prospective questionnaire, which sought to find the ability to anticipate the performance of students in the task, and the retrospective questionnaire, which is aimed at establishing a self-perception of student performance (Sandi-Urena et al., 2012; Frumos, 2015; Young and Worrell, 2018). The classification of the answers to the questionnaire was done by creating subcategories related to the level of development of metacognitive regulation activities, based on the declarative knowledge of students, i.e., their self-perception of the task.

**Table 5.** Test designed for this research with the topics about the concept of the tangent line problem and metacognitive regulation processes in each phase.

| Sub-categories | Definition of the Sub-category | Initial Origin Category in Classification | Category where the strategy appears in the questionnaires |
|---|---|---|---|
| DCT | Declare full understanding | Planning | Planning-Control or Monitoring |
| DCP | Declare partial understanding | Planning | Planning-Control or monitoring |
| DFC | Declaring failures in comprehension | Planning | Planning- Control or Monitoring |
| ORV | Observe relationships between variables | Planning | Planning |
| EDT | Establish the demands of the task | Planning | Planning |
| EPA | Develop an action plan | Planning | Planning |
| TRO | Transfer from one representation to another | Planning | Planning-Control or Monitoring |
| IRE | Required Information not found | Control or Monitoring | Evaluation |
| FEE | Using external sources for information search | Control or Monitoring | Planning-Control or Monitoring-Evaluation |
| ACP | Activating prior knowledge | Control or Monitoring | Planning-Control or Monitoring |
| SCI | Selection and classification of information | Control or Monitoring | Control or Monitoring-Evaluation |
| URT | Use of technological resources | Control or Monitoring | Planning-Control or Monitoring |
| MTT | Time management to develop the task | Control or Monitoring | Planning- Evaluation |
| AMP | Adaptations and modifications to the problem | Control or Monitoring | Evaluation |
| RSA | Reflections on learning | Evaluation | Control or Monitoring Evaluation |
| Dde | Error and difficulty detection | Evaluation | Evaluation. |
| OIC | Observed inconsistencies or confusion | Evaluation | Evaluation |
| VSH | Verification of solutions found | Evaluation | Evaluation |

Source: Prepared by the authors.

From this classification in the prospective questionnaire, it was possible to identify that only 9% of students declared that they fully understood the task; about 36% indicate that they have understood the task only a little and 55% maintain that they have understood the main elements. Regarding the retrospective questionnaire, 21% of students declare that they have completed the task, 16% only developed the task a little bit, 32% solved some elements of the task, and 31.58% declare that they could not solve it or their solution was not satisfactory.

Besides the analysis of the prospective questionnaire, the responses of the students to the retrospective questionnaire were classified, Table 5 shows those results.

The purposes of using this type of instrument are to make the students reflect on their own learning processes and to make them aware of their learning difficulties (Elif et al., 2011; Zubaidah, 2016; Braund, 2017; Moshman, 2018, Veenman and Van Cleef, 2019; Hashemi et al., 2015; Lan and Ying, 2021; Vega et al., 2014; Wewe, 2020; Zambrano et al., 2019; Zengin, 2018).

### 3.2. Statistical Analysis of the Prior Knowledge Test

Table 6 shows the results of the prior knowledge test of both groups, in relation to competencies: solution of equations, identification of terms and conceptual questions (SE, IT and C).

**Table 6.** Prior knowledge test results.

| Type | Group | Number of students | Average | Typical deviation | Shapiro-Wilk Statistician | Degrees of freedom | Significance |
|---|---|---|---|---|---|---|---|
| IT IS | Group 1 | 18 | 76,74 | 11,9 | 0,888 | 18 | 0,036 |
| | Group 2 | 21 | 78,9 | 5,26 | 0,915 | 21 | 0,068 |
| IT | Group 1 | 18 | 62,33 | 5,52 | 0,89 | 18 | 0,039 |
| | Group 2 | 21 | 61,46 | 5,07 | 0,89 | 21 | 0,022 |
| C | Group 1 | 18 | 72,22 | 9,39 | 0,951 | 18 | 0,446 |
| | Group 2 | 21 | 69,52 | 5,28 | 0,958 | 21 | 0,474 |
| Total | Group 1 | 18 | 211,29 | 19,94 | 0,964 | 18 | 0,685 |
| | Group 2 | 21 | 209,89 | 10,34 | 0,943 | 21 | 0,251 |

Source: Prepared by the authors

By group, the normality of the data can be seen according to the Shapiro-Wilk statistics (p> 0.05). Table 7 shows the hypothesis tests of prior knowledge test for SE, IT and C categories (p>0,05).

**Table 7.** Verification of Variance.

| Prior knowledge | Verifications of Variances | Levene's test for equal variances | | T-test for average equality | | |
|---|---|---|---|---|---|---|
| | | F | Gis. | T | Degrees of freedom | Gis. (bilateral) |
| SE | No equal variances have been assumed | 14,083 | 0,000 | -0,72 | 23 | 0,480 |
| IT | Equal variances have been assumed | 0,32 | 0,580 | 0,51 | 37 | 0,610 |
| C | Equal variances have been assumed | 1,794 | 0,190 | 1,13 | 37 | 0,270 |
| Total | Equal variances have been assumed | 3,272 | 0,080 | 0,28 | 37 | 0,780 |

Source: Prepared by the authors

Initially, equal variances were tested to identify the T statistic of the mean difference test. Only different variances were found in the SE category, however, no differences between the means were found in the T-test, which indicates the possibility of a statistically homogeneous population for procedural competencies in mathematics. Therefore, the assignment of the experimental group and the control group was done freely.

### 3.3. Pretest - Posttest group related differences

The intervention showed that the average differences in the results expressed as a percentage in Figure 2, between the Pretest and the Posttest of the experimental group, were around 13.64 percentage points. Consequently, the virtual laboratory is facilitating processes of self-regulation around the differential calculation course, explicitly in the study from the tangent line problem approach.

Likewise, Figure 3 shows that, according to the results of the average percentage, the experimental group (***mean = 77.27***) is slightly better than the control group (***mean = 69.52)***. In addition, a constant and parallel growth of the experimental group with the control group is observed, which indicates that it is an alternate form of learning and produces results similar to traditional teaching. These slight differences (7.78) between the means of the experimental and control groups can be explained from differences in category C in the Pretest. Additionally, Table 8 shows that in the Pretest, there were no differences in SE, while in the Posttest some effects were identified.

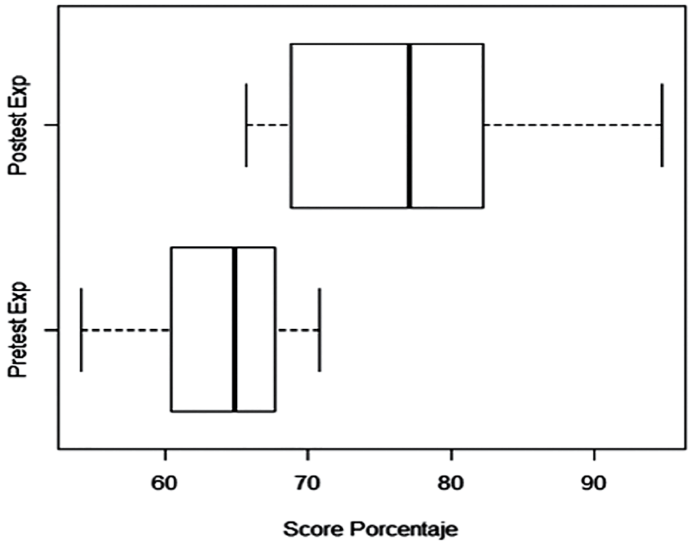


**Figure 2.** Comparative Pretest vs. Posttest Experimental Group
Source: Prepared by the authors



Table 8 shows statistically similar variances between both groups, which allows us to assume homogeneous populations within each one. Additionally, the homoscedasticity data do not have biases concerning the population, since Levene's tests only show differences, in (*), (**), (***).

**Table 8.** Pretest-Posttest group statisticians.

| Group statistics | | | | Levene's test for equal variances | | T test (joint) for average equality | |
|---|---|---|---|---|---|---|---|
| Variable | Group | Average | Typical deviation | F | Sig.*+ | t (GL-25) | Gis. (bilateral) |
| C(pre) | Experimental | 76,45 | 12,21 | 0,51 | 0,482 | 2,553 | 0,017* |
| | Control | 63,84 | 13,36 | | | | |
| IT(pre) | Experimental | 58,88 | 7,21 | 0,885 | 0,356 | 0,74 | 0,466 |
| | Control | 56,79 | 7,50 | | | | |
| SE(pre) | Experimental | 52,15 | 2,41 | 0,197 | 0,661 | 0,679 | 0,504 |
| | Control | 51,49 | 2,64 | | | | |
| C(pos) | Experimental | 78,77 | 11,97 | 1,409 | 0,246 | 0,437 | 0,666 |
| | Control | 76,98 | 9,29 | | | | |
| T(pos) | Experimental | 75,41 | 8,52 | 0,751 | 0,395 | 1,587 | 0,125 |
| | Control | 70,36 | 8,02 | | | | |
| SE(pos) | Experimental | 69,62 | 7,96 | 2,862 | 0,103 | 3,386 | 0,002*** |
| | Control | 60,75 | 5,52 | | | | |
| Total(pre) | Experimental | 189,17 | 14,04 | 0,001 | 0,979 | 2,893 | 0,008** |
| | Control | 172,11 | 16,40 | | | | |
| Total(pos) | Experimental | 219,98 | 13,35 | 0,743 | 0,397 | 2,083 | 0,048* |
| | Control | 208,08 | 16,07 | | | | |

Note: *+Equal variances (p>0.05); **p<0.05; ***p<0.01; p<0.005;
Experimental Group (n. 13); Control Group (n-14)
Source: Prepared by the authors

The differences identified in the Pretest are related to conceptual (C) competencies, specifically in the students of the experimental group. For the Posttest, there were differences in the SE category in the experimental group, which was evidenced in the tests of the inter-subject effects (analysis of covariance), related in Table 9, specifically in the SE- Posttest category, which shows the influence of the intervention in the experimental group, with an effect of 30.3% of the variability explained within the sources of design variation.

Consequently, there were improvements in the SE category's competence, therefore, there were also differences in the experimental group, with respect to the control group

**Table 9.** Testing of inter-subject effects (Covariance Analysis). Dependent variable: SE(pos).

| Origin | Sum of type III squares | Gl | Quadratic average | F | Gis. | Eta to partial square |
|---|---|---|---|---|---|---|
| Model corrected | 620,479a.m. | 2 | 310,239 | 6,981 | 0,004** | 0,368 |
| Intersection | 41,253 | 1 | 41,253 | 0,928 | 0,345 | 0,037 |
| Group | 464,096 | 1 | 464,096 | 10,443 | 0,004** | 0.303++ ++ |
| SE(pre) | 90,145 | 1 | 90,145 | 2,028 | 0,167 | 0,078 |
| Error | 1066,539 | 24 | 44,439 | | | |
| Total | 115832,23 | 27 | | | | |
| Total corrected | 1687,018 | 26 | | | | |

Note: a. R squared ,368 (corrected R squared ,315). Slightlyne's contrast on the equality of error variances. F(1,25)-1;025.p-0.310. (Homoscedasticity).++ Posttest effect of 30.3% of explained variability.** $p<0.005$
Source: Prepared by the authors

## Virtual Lab

The simulations presented by the students showed that they identify the behavior of the functions and recognize the concept of approximation from the left or the right at a point for a given function. Furthermore, the graphs used in the virtual laboratory allow the students to visualize the behavior of the functions and their respective derivatives, as well as the tangent lines to the original function.

From the development of the task, it was observed that:

1. The students identify and group the points at which tangent lines are presented with positive, negative, and near-zero slopes.
2. The students establish relationships between slopes to identify the largest and the smallest slopes.
3. The students understand the difference between a zero slope and an infinite slope.

## Conclusions

Exposing students to situations of self-reflection on their learning processes allows them to examine their judgments on the way they develop the task, at different moments of the teaching-learning process.

Using the declarative knowledge of the students, that is, their perception of what they say or do, to develop the task, allows us to identify if their judgments about the actions to carry out the task are adjusted to their real performance or if they are inconsistent with the task resolution process. Indeed, most students maintain that they acquire very little understanding of the problem, the demands of the task, and the main elements that may be the key to solving the problem, which is consistent with the general domain hypothesis about performance on a test.

Likewise, we observe that there is a combination of strategies from different categories and, specifically, we analyze subcategories related to poor understanding of the task, such as DCP and DFC that originally appeared in the planning category, but also occurred in the control and monitoring category. Subcategories such as ACP and URT, related to the activation of prior knowledge and the use of technological resources, which are initially classified within monitoring categories, also migrate to the planning category, in short, the subcategories are used in a certain way by students without any discrimination or differentiation around the processes of metacognitive regulation.

The evaluation category showed three subcategories that remain invariant: DDE, OIC, and VSH, related to the detection of errors and difficulties. Inconsistencies or confusions were observed and the solutions found by the students when developing the task and essential to the evaluation category were verified.

Finally, the analysis of the subcategories shows that the level of students to develop metacognitive regulation strategies is low and that the poor understanding of the problem and the demands and fundamental elements of the task is the most recurrent points around the student declarations. In addition, limited knowledge of the different ways of regulating their learning is observed, which implies little effectiveness in the development of the task.

## Limitations and Further Studies

We are clear that self-report type measures, such as those in which students declare something about their own knowledge, in most cases, are not very informative if they are not accompanied by measurements of their performance, which allows checking whether there is coherence between what they say and what they do. Therefore, we must study this type of relationship in future works of research to establish whether, metacognitively speaking, there are significant changes in learning. Similarly, the calibration problems of the metacognitive judgments made by the students in future research are also suggested.

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