Truth-Making and the Alethic Undecidability of the liar

Los hacedores de verdad y la indecidibilidad alética del mentiroso

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Resumen

En el presente artículo, sostengo que es posible una nueva solución a las paradojas semánticas, basada en los hacedores de verdad. Demuestro que, con base en una comprensión adecuada de cuáles son los últimos hacedores de verdad o falsedad de las oraciones, se puede demostrar que oraciones como el mentiroso son aléticamente indecidibles. Esto significa que no se puede decir en principio si tales enunciados son verdaderos, no verdaderos, falsos, no falsos, ni verdaderos ni falsos, verdaderos y falsos, y así sucesivamente. Sostengo que esto conduce a una solución de las paradojas semánticas que parece estar libre de los problemas de venganza, nos permite conservar la lógica clásica y la validez del esquema-T.

Palabras clave

indecidibilidad alética, indeterminación, el mentiroso, venganza, verdad, condiciones de verdad, verdad de las decisiones, esquema-T.

Abstract

I argue that a new solution to the semantic paradoxes is possible based on truth-making. I show that with an appropriate understanding of what the ultimate truth and falsity makers of sentences are, it can be demonstrated that sentences like the liar are alethically undecidable. That means it cannot be said in principle whether such sentences are true, not true, false, not-false, neither true nor false, both true and false, and so on. I argue that this leads to a solution to the semantic paradoxes that appears to be free of revenge problems, allows us to maintain classical logic and the validity of the T-schema.

Key words

alethic undecidability, indeterminacy, Liar, revenge, truth, truth-conditions, truth-making, T-schema.

This paper sketches a new solution to the Liar and other semantic paradoxes based on the phenomenon of truth-making. According to to the solution sentences like the Liar are undecidable with respect to all their alethic properties. That means it's undecidable—it cannot be said in principle—whether such sentences are true, false, either true or false, neither true nor false, both true and false, and so on. This *alethic undecidability* is not due to lack of information or some verification transcendent fact concerning such sentences. Rather, as I shall argue, it's an objective fact about these sentences that arises from their meanings, their referential properties, and the nature of truth itself. The aspect of the nature of truth that concerns us is that the facts of truth (and falsity) are *dependent facts*, that is, that a sentence *S* is true, or that *S* is false, etc, is a fact that is made the case by reality, and so is never a basic fact. In other words, truths are made. I will argue that truth's dependency in this sense, given facts about the referential properties of paradoxical sentences, entails that such sentences are alethically undecidable, and that this is the basis of a solution to the semantic paradoxes. Call this the *TM-solution* (truth-making solution). The TM-solution allows us to perserve classical logic, or at least involves no revision of logic as such and places no limitation on the expressive power of a language. What it entails is that we rethink the theoretical role and nature of truth-conditions, in relation to linguistic-meaning and reference, inference, and validity.

The paper proceeds by articulating the basic truth-making principle, ***TM***, and the concept of alethic undecidabilty §(I). In §II, we look at the class of *ground-unspecifiable* sentences, which are sentences that exhibit certain looping or infinitely descending referential chains. This class includes, but is not exhausted by, paradoxical sentences. I show that by appealing to ground-unspecifiable sentences, an apparent refutation of ***TM*** becomes available. In §III, I show that we can rebut the refutation of ***TM*** by recognition of a specific idea of what truth-conditions are, given ***TM***. I show in detail how this works in §IV. Finally in §5, we assemble the results accumulated in the course of paper, which deliver a solution to the semantic paradoxes.

I
Truth-making and alethic undecidability

The first two concepts that we need to be acquainted with, and which play a crucial role in what follows, are truth-making and alethic undecidability. Here is the core contention regarding truth-making.

Truths (and falsities) are made true (made false) by how things are. Truth and falsity is dependent. Facts of truth are never basic facts. So, a sentence *Snow is white* is true in virtue of the fact that snow is white. Where *S* is *snow is white*, the fact that *S is true* is true, ultimately, in virtue of the fact that snow is white. Likewise, the fact that *Snow is pink* is false holds in virtue of the fact that snow is white. The fact that snow is white grounds a vast hierarchy of sentences.

Here is a fragment of that hierarchy of sentences, all grounded in that fact about snow:

etc.	etc
(2) (1) is true	(2′) Either (1′) is true or (0′) is true.
(1) (0) is true	(1′) (0′) is true
(0) Snow is white.	(0′) Snow is not white

Snow is white

There is no limit on the logical complexity of the sentences in the hierarchy. The truth and falsity of all the sentences in this infinite hierarchy are grounded, in the sense that they have truth-makers or false-makers.[1]

We now look at an important distinction regarding *grounding-facts*. In the hierarchy above the grounding-fact is a *non-alethic fact*, viz., a fact that has nothing to do with truth and falsity. The fact that snow is white is not a fact about a proposition's being true or false. We can allow that non-alethic reality includes facts of meaning and reference, and use, facts about speech acts, and so on, that is, linguistic and semantic facts excluding facts of truth and falsity. Now consider all non-alethic facts, in this sense, in one vast super fact. Then, a vast hierarchy of sentence truth and falsity will be determined by that fact.

There are questions, with respect to the metaphysics of truth-making and this great hierarchy. For example should non-alethic fact include negative facts, like snow's not being black, or totality facts, such as that all the objects in a certain position are white, and so on? Or indeed, should we think of the entities that make sentences true/false as facts, or indeed, as

[1] I assume in what follows that both sentences (given that they express thoughts) and thoughts themselves, or propositions, can be truth-bearers.

entities at all? I am sweeping all these issues aside here, as more or less, orthogonal to our concerns in this paper. (See Barker and Jago (2012) for ideas about what can be said regarding some of these issues.) In what follows, I assume for convenience, no logical restrictions on the kinds the facts there are, though nothing hangs on this for my purposes below.

By *grounding-fact* we mean any fact that can be amongst the ultimate truth/falsity-makers for some given sentence. We have seen that grounding-facts must include non-alethic facts, that is, facts that have nothing to do with truth and falsity of sentences or propositions, like the fact that snow is white. The question now is: do grounding facts include some alethic facts? The answer is *yes*. Some alethic facts are grounding facts, though not all; the vast majority are not—they are non-basic. Consider sentences about truth-making, like:

S^T : The sentence *Snow is white* is true in virtue of the fact that snow is white.

What makes S^T true is not just the non-alethic fact that snow is white, though this plays a role. What S^T describes is a relation between a fact of truth and the fact that snow is white. That relation holds because of the nature of truth itself. It's a necessary fact about truth that the truth/falsity of sentences/propositions are dependent. This is the fact that truth (and falsity) is *inherently dependent*. The complete truth-maker for S^T is then:

(a) Snow is white.
(b) Truth is inherently dependent.

S^T's truth-maker is a mix of non-alethic fact, and the metaphysically necessary fact of truth's dependency. The fact of truth's dependency can itself function, all by itself, as a truth-maker. For example, the truth that all truths are dependent is itself made true by this fact about the nature of truth. Search in vain and you will not find the truth-maker of this sentence amongst the non-alethic facts.

Given truth is inherently dependent, what makes sentence true/false will be non-alethic fact or the fact of truth's dependency. The alethic facts that are not in the class of ultimate grounding facts are the facts that simply correspond to the facts of truth and falsity. These are facts like: *The fact that it's true that snow is white*, or *The fact that it's true that it's true that it's true that snow is white*, and so on. All these sorts of alethic fact are non-basic. They are not grounding-facts, but rather, grounded facts.

We can now sum up the view about truth (and falsity) being proposed:

TM: If a sentence *S* is true/false, then it's true/false, ultimately, in virtue of non-alethic fact or the fact that truth/falsity is inherently dependent.

People may take ***TM*** to be a heavy duty metaphysical thesis. I don't think it is. It's just that metaphysicians make heavy weather of it. However, as already indicated, I am not concerned with these storms here. In particular, I leave aside issues as to whether there really are negative facts or totality facts, or facts at all. All we need is the bare idea that truth (falsity) is dependent.

Nevertheless, there is another issue about how to understand ***TM***. How are we to think about the truth-making relation? I take truth/falsity-making to involve a *grounding* or *in-virtue-of* relation between a proposition's truth or falsity and facts. Grounding is an asymmetric determinative relation—Rodriguez-Pereyra (2005). The form of a truth-making fact, then is this, where *X* is some fact and *S is true* is the condition or state of affairs that *S* is true:

$$X \text{ ==> } S \text{ is true.}$$

This construal of truth-making assimilates truth-making to a more general phenomenon of grounding. For example, falsity is in as much need of grounding as is truth. So facts of falsity are grounded, which means, there are facts like this:

$$X \text{ ==> } S \text{ is false.}$$

This way of thinking about truth/falsity-making makes sense. Facts of truth and falsity are grounded in the same way that facts of personal identity, or facts mental life, and so on, are grounded.[2] Grounding is a relation between facts. Again, what follows does not depend precisely on this way of understanding truth-making, but it's a convenient way of thinking of it for our purposes.

[2] See Barker (2012) for development of these ideas. This way of understanding things is at odds with the standard idea that truth-making is a primitive relation, *truthmaking*, which links a fact and proposition (see Armstrong 2005.) This approach ignores false-making. We could develop the argument below using this understanding, but it's more complicated.

Finally, a quick note on facts: I take facts to be the grounders of truth and falsity. Again, this will disturb some metaphysicians. I take facts to be *obtaining conditions*, where a *condition* is a state (*possibilium*) that may or may not obtain. A certain realism about facts and states of affairs makes the formulation of arguments much simpler.

Alethic undecidability

We now look at the thesis of *alethic undecidability*. Consider the Liar, *L*, which says of itself that it's not true. According to the TM-solution, *L* is alethically undecidable. That means we cannot say in principle that *L* is true. We cannot say, in principle, that *L* is false. Nor can we affirm that *L* is bivalent, viz., that it is either true or false. However, our inability to affirm bivalence is not the result of our affirming a truth-value gap. On the contrary, according to the TM-solution, we cannot affirm that *L* lacks truth-value, that is, that it is neither true nor false. So, the indeterminacy we are proposing is not the familiar truth-value gap indeterminacy, proposed by Kripke (1975), Goldstein (1999), Field (2007), Maudlin (2007). Since we cannot affirm that *L* is true and cannot affirm it's false, we cannot affirm that it is both true and false, that is, we cannot affirm that *L* suffers a truth-value *glut*. So, Priest's (1987) dialetheic position cannot be affirmed.

Let's try and fix the ideas in play here a bit more precisely. Let an *alethic predicate* be any one-place open-sentence of the form *(…x …true…)* or *(…x …false…)*, where the other constituents of the open-sentence are standard logical operators: '&', '∃*x*' and '¬', etc. So, alethic predicates include, *x is true*, *x is false*, (¬¬¬*x is true*), and so on. Alethic predicates do not include certain intentional verbs like *believe* or *say*. So, (*believes of x that x is true)*, *(x is said to be true)*, or *(x cannot be said to be true*) are not alethic predicates. Call an *alethic property* any property corresponding to an alethic predicate. We shall be relaxed about the metaphysics of properties in what follows and allow that each meaningful predicate corresponds to a property. Again, nothing hangs on this except convenience of exposition.

That's the concept of alethic property. Before we proceed, we need to know more about the sense of *cannot-say* being used here. Let assertion be a speech-act whose only norm is to represent how things are. That's not an implausible idea about assertion. Perhaps there are other norms, like knowledge, or certainty, etc, but I am not concerned with those here, nor are they required for my argument. Let's suppose we have a body of information ***I*** that is perfectly informative about non-alethic reality and inherent dependency of truth, and about the reference and linguistic meaning of a sentence *S*. There is no epistemic constraint on ***I*** whatsoever. Let *P* be any alethic property, such as being true, being false, being either true or false, being neither true or false, and so on. What I am proposing is the following conception of alethic undecidability:

> *Alethic undecidability*: An ideally functioning assertor, furnished with perfect information ***I***, cannot assert that *S has P*, for whatever alethic property *P*.

This conception of alethic undecidability does not involve accepting some kind of anti-realism about alethic reality. There are no epistemic constraints on ***I***. It does, however, involve accepting that alethic properties of a sentences *S* are dependent on *S*'s meaning and reference, and facts about the world. That's uncontroversial. For surely, a given sentence's truth does depend on what it means, and the facts, out there.

Another, related, way of understanding alethic undecidability is this. A perfect representation of reality will have a gap in it with respect to the alethic status of L. It will lack the representations: *L is true*, *L is false*, *L is either true or false*, *L is neither true nor false*, *L is both true and false*, *It's not the case that L is not true*, and so on. This is not because our language cannot express something about its truth-predicate. We assume it's as powerful as a language can be. We also assume that the perfect presentation reflects perfectly non-alethic reality. There is no epistemic restriction or verification transcendent fact. In short, the ideal representation has a *representation gap*, but there is no truth-value gap. The representation gap is the fact that in the ideal representation there is a gap with respect to the alethic status of *L*. In contrast, for the truth-value gap approach, there is no representation gap, since the ideal representation will represent *L* in these terms: *L is neither true nor false*, which is the representation of an alethic property. That's not present in the ideal representation, according to the TM-solution.

The TM-approach is not the proposal of an intuitionist logic, in which, say, we might affirm that ¬¬(*L is true*) but not be able to infer *S is true*. To affirm ¬¬(*L is true*) is to fill the representation gap about *L*. It's to represent *L* lacking the property of lacking truth. But given alethic undecidability, the ideal representation will lack this representation as well.

Below, phrases like *One cannot say that S is true*, and so forth, will be used to express this fact about an ideal assertor, given information ***I***. In saying that it's undecidable what *S*'s alethic properties are we mean just this: we cannot say what its alethic properties are. Given that we cannot affirm (in principle) a gap in the case of the Liar, *L*, the familiar revenge problem for the truth-value gap approach cannot arise for the TM-approach. We cannot say that *L* is neither true nor false, and so, we cannot say that *L* is not true. Whereas, on the truth-value gap approach, we must say that *L* is not true (as a commentary claim about *L*.) Therefore, we are committed to saying, by the T-schema, that *L* is true, after all. However, we cannot say this on the TM-approach.

You might object that there will be other forms of revenge that can threaten this position. And indeed there are forms of apparent revenge that we must consider below. Take the predicate: *cannot be said to be true*. This is not an alethic predicate so doesn't correspond to an alethic property. However, there is a seeming revenge using it: a sentence that says of itself that it cannot be said to be true. We examine this case below in §V.

How then does ***TM*** generate this alethic undecidability? To answer that we need to move to our next topic: the class of sentences that exhibit what I call *ground-unspecifiability*.

II
Ground-unspecifiability and the refutation of TM

My case for the alethic undecidability of the liar is based on rejection of an attempted refutation of ***TM***. I outline this argument below. The refutation involves appeal to the properties of a specific class of sentences, which I call *ground-unspecifiable sentences* (for reasons to that will become clear below.) Semantically paradoxical sentences are members of this class of sentences. I now explain the property of being ground-unspecifiable, and then display the refutation of ***TM*** that's based on reasoning about this class of sentences.

Consider the sentences below. *L* is a semantically paradoxical sentence: a Liar. The other sentences, *Infinite Descent* and *T*, are *truth-tellers*. These sentences are not considered paradoxical. Nevertheless, they are in the same class that I am calling *ground-unspecifiable sentences*:

Infinite Descent: (0) (1) is true
(1) (2) is true
(2) (3) is true
(3) (4) is true
(4) (5) is true…
etc

Looping:
T : T is true.

L : L is not true.

These sentences are perfectly meaningful in the sense that they are grammatical, have meaningful predicates, and suffer no apparent reference failure. What each exhibits is a specific loopy/infinitely descending referential chain. We might put it his way: if you follow the reference of these sentences around, or down, to determine what they are about, then, you always encounter a specification of a condition about the truth, lack of truth, falsity, or lack of falsity of a sentence, and so on.

Let's refine this idea. Take *Infinite Descent*. The sentence (0) says *(1) is true*. (1) says *(2) is true*. (2) says *(3) is true*, and so on. Each sentence in the list simply says that the sentence below it is true. At no point do we find a sentence that says something about non-alethic reality—like *snow is white*, or *grass is purple*—or about the nature of truth, such as it's dependent. In short, no sentence in the list specifies a *grounding-condition* of the kind required by ***TM*** for truth or falsity, even though there is no reference failure or meaningless predicate. Similar comments apply to T and L. In sum: The linguistic and referential facts about these sentences don't determine a specification of a grounding-condition, whose obtaining or non-obtaining could be said to make them true or false. That's the property of being *ground-unspecifiable*.

Being *ground-unspecifiable* is about not determining any *specification* of a possible grounding-fact. Being *ungrounded* is the distinct property of actually lacking a grounding-fact: there is no grounding fact that makes the sentence true or false. The main premise in the argument threatening ***TM*** links these two properties thus:

Kripke's Thesis: If a sentence is ground-unspecifiable, it's ungrounded.

One can discern *Kripke's Thesis* in Kripke's (1975) theory, hence the name of the principle. I won't justify that interpretation here.

The refutation of ***TM*** that I mentioned above as threatening ***TM***, has this form—here S is a ground-unspecifiable sentence:

TM-Refute

1. Suppose ***TM***.
2. S is ungrounded. [By *Kripke's Thesis*]
3. S is not true. [By ***TM*** from *2*.]
4. Sentence *step 3* above is true. [By the *T*-schema, from *3*.]
5. *Step 3* is ground-unspecifiable and so ungrounded. [By *Kripke's Thesis*.]
6. *Step 3* is not true. [By ***TM*** and *5*.]
7. *Step 3* is true and not true. [From *4* and *6*.]
8. ***TM*** is false. [Reductio *1-7*].

Note that the sentence *step 3* is a ground-unspecifiable sentence because it's a truth-claim about a ground-unspecifiable sentence *S*.

To conclude: ***TM*** looks like it's refuted, given *Kripke's Thesis*. Some might say: so much the worse for ***TM***! But I think ***TM*** is a correct principle about truth. I want to demonstrate how we can save it. My way of doing so is by questioning *Kripke's Thesis*. Observe, however, that this might, at first, look like a hopeless strategy.

If we deny *Kripke's Thesis*, we must find a counterexample to it: a ground-unspecifiable sentence *S* (one like *Infinite Descent*, *T* or *L* above) that, nevertheless, has a grounding-fact, that is, a truth-maker of the kind required by ***TM***. But how could we find any such thing? Take (0) of *Infinite Descent*. None of the sentences beneath it, upon whose truth it would depend if it were true, describes any grounding-condition, like *snow is white*. So, how can we link (0), in the end, to a grounding-fact as its truth/false-maker? We cannot. The moral seems to be general: No ground-unspecifiable sentence *S* can be associated with a grounding-fact as its truth/falsity-maker. So it seems we cannot deny *Kripke's Thesis*.

That might suggest my strategy for saving ***TM***—questioning *Kripke's Thesis*—is hopeless. But not so. It's here that the twist of this paper comes in. My position isn't that we should deny *Kripke's Thesis***.** Rather, it's that *Kripke's Thesis* is alethically undecidable: it's essentially undecidable whether it's true, false, either true or false, and so on. If *Kripke's Thesis* is alethically undecidable, we cannot use it in ***TM-Refute***, since we cannot affirm it.

The challenge for this solution is to find an independent reason for thinking that *Kripke's Thesis* is undecidable. The rest of this paper provides that independent reason.

III
Truth-conditions and inference

The key is how to think of sentence truth-conditions given ***TM***. If you accept ***TM*** you ought to hold that truth-conditions are linked to *possible truth-makers*. This means that if a sentence has truth-conditions, then, given reference and meaning, there is a *condition* that would, if it obtained, make it true, and another condition that, if it obtained, would make it false. Call this principle ***TC***. ***TC*** is not a standard idea about truth-conditions, but, I will argue, it goes with ***TM***. Given ***TC***, we can show that the sentence *S is ungrounded*, that is, *step 2* in ***TM-Refute***, cannot be said to have truth-conditions or not. Its truth-conditional status is essentially undecidable. If so, the truth-conditional status of *Kripke's Thesis* is undecidable. The argument against ***TM*** is then blocked, but not by denying *Kripke's Thesis*, but by affirming its undecidability.

In short, *Kripke's Thesis* goes beyond what can be said, not because our language has some expressive deficiency, but because of the nature of truth-conditions themselves, given ***TC***. Once that's established, we get the result that we can keep ***TM***, but ground-unspecifiable sentences, like *Infinite Descent*, *T* and *L* are alethically undecidable. Consequently, the Liar, *L*, and its kin, are alethically undecidable. This leads to a new kind of solution to the Liar and other semantic paradoxes: alethic undecidability.

Let's examine ***TC*** more closely. By *grounding-condition* we mean the kind of condition whose obtaining (as a fact) is, by ***TM***, an ultimate truth/false-maker. The phrases *x => S is true* and *x => S is false*, as we noted above, signify, respectively, that *x*'s obtaining makes it that *S* is true and *x*'s obtaining makes it that *S* is false. ***TC*** then is:

TC*:* *S* has truth-conditions iff there is a grounding-condition *x* such that if *x* obtains, *x* => *S* is true and a grounding-condition *y* such that if *y* obtains, *y* => *S* is false.

Since the conditions *x* and *y* are not themselves *facts* but states that may or may not obtain, ***TC*** does not imply that a given sentence *S* is bivalent. Both *x* and *y* might fail to obtain. I note again that there is nothing particularly metaphysically loaded about any of these ideas. The metaphysical posits in this paper are merely convenient ontology for expressing the core ideas directed towards semantic matters.

My purpose is to show that ***TC*** is very plausible given ***TM***. Of course, ***TC*** is not a standard idea about truth-conditions. Truth-conditions, in standard semantics, are associated with the biconditionals, like *S is true iff P*, where *P* expresses the content of *S*. This is a *correspondence conception* of truth-conditions. It's this that we cannot accept if we accept ***TM***. I want to indicate how this is so.

Let's think of a truth-condition for a sentence *S* is a condition *C*, that may or may not obtain, under which *S* is true. It's the condition described by *S*. The condition *C* that is the truth-condition of a sentence *S* is either alethic or non-alethic. If *C* is non-alethic, then, by ***TM***, if *C* obtains, it must be the truth-maker for *S*. If *C* is alethic, then, by ***TM***, if it obtains, there must be a non-alethic condition *x*, in virtue of whose obtaining *C* obtains, and in virtue of which *S* is true. This holds likewise for the false-condition for *S*. We have just shown that if *S* has truth-conditions, then the right-hand side of ***TC*** holds. The converse also holds (obviously).

One might object that a ground-unspecifiable sentence like (0) in *Infinite Descent*, has truth-conditions, namely, that *(1) is true*. That is, we express these conditions thus:

$$\mathrm{T}^{(0)} : \text{(0) is true iff (1) is true.}$$

In other words, one might contend, that (0) is associated, by virtue of its meaning and reference, with a *condition*, a state of affairs, that if it obtains, will necessitate (0)'s truth, and such that, if (0) is true, must obtain. Surely then (0) has truth-conditions. But note we cannot associate any non-alethic condition with (0). And so, this is not in accord with ***TC***.

This objection should not affect the proponent of ***TM***. The objector's idea of what it is to have truth-conditions involves a biconditional, $T^{(0)}$. To affirm that (0) has as its truth-conditions, we have to affirm this biconditional, but that requires we can make sense of supposing that (1) is true. But, if (1) is true, then (2) must be true, and so on ad infinitum. In short, if (1) is true, then an infinite chain of facts of truth must underlie its truth, but we cannot say, at any stage, that there is any non-alethic condition in the chain of facts descending downward. But affirming this is clearly at odds with affirming ***TM***. If so, you cannot accept ***TM*** and accept the bi-conditional conception of truth-conditions, which is the standard view. So what view ought you to accept? I say ***TC***.

Inference and truth-conditions

Below I show that given ***TC***, we can conclude that it's undecidable whether sentences like *step 2 in* ***TM-Refute*** have truth-conditions or not. It follows from this, I suggest, that we cannot use *step 2*, and similar sentences, in inferences. In drawing that particular conclusion, I am invoking a regulative principle governing inference:

Inference-Rule: If it's undecidable whether a sentence *S* has truth-conditions or not, then we ought not to use *S* in logical inference.

Inference-Rule looks totally plausible. Indeed, if you cannot say a sentence has truth-conditions or not, you ought not to use the sentence in inference. Of course, you might use such a sentence *S* because you are ignorant of this alethic undecidability. Or you might explicitly assume you can affirm *S*'s having truth-conditions. That's consistent with *Inference-Rule*. But once you uncover a contradiction based on that assumption, then in withdrawing the assumption, you should cease using *S* in logical arguments.

IV
Saving TM and Semantic Entanglement

We have assembled all the required elements to block the argument ***TM-Refute***, which threatens ***TM***. Let's see how this works. ***TM-Refute***, recall, concerns ground-unspecifiable sentences, like *Infinite Descent* (0) and *Looping T*, above. The premise in the argument that concerns us is *Kripke's Thesis* and *step 2*. *Kripke's Thesis* amounts to accepting the inference:

> ***K***: *S* is ground-unspecifiable. Therefore, *S* is ungrounded.

It's this inference we are questioning. My contention isn't that ***K*** is invalid. It's that ***K***'s validity is *undecidable*. That's why *Kripke's Thesis* is alethically undecidable. If so, one cannot use ***TM-Refute*** against ***TM***, since it's main premise, *Kripke's Thesis*, is undecidable.

The argument for undecidability of ***K***'s validity focuses on the sentence *step 2*, which is the conclusion of ***K***: *S is ungrounded*. My argument is that given ***TC***—our principle about truth-conditions—it's undecidable whether *step 2* has truth-conditions or not. If the premise of ***K*** is true, but we cannot say whether its conclusion, *step 2*, is true or false, then we cannot say whether ***K*** is valid or not.

I now show that *step 2*'s truth-conditional status is undecidable given ***TC***. *Step 2* is the claim that no grounding-fact makes *S* true or false. We can represent *step 2* more perspicuously as below:

Step 2: $\neg\exists x$(x is a grounding-fact & x => *S* is true).[3]

Note now three things about *step 2*:

> *(i)* The sentence *S is true* is a constituent of *step 2*.
> *(ii) S is true* is a ground-unspecifiable sentence.
> *(iii)* If it's undecidable that *S is true* has truth-conditions, it's undecidable that *step 2* has truth-conditions.

Facts *(i)* and *(ii)* are obvious. Let me argue for *(iii)*. Say it's undecidable that *S is true* has truth-conditions. Then, it's undecidable, evidently, that the open-sentence,

x => *S* is true,

has *satisfaction-conditions*. If so, it's undecidable, obviously, that the open-conjunction,

(x is a grounding-fact & x => S is true),

has satisfaction-conditions. Then it's undecidable, clearly, that the existential quantification,

$\exists$*x(x is a grounding-fact & x => S is true*),

has truth-conditions, and therefore, it's undecidable that its negation, *step 2*, does.

What I show now is that, given ***TC***, it's undecidable that *S is true* has truth-conditions. In which case, it's undecidable that *step 2* has truth-conditions, and consequently, it's undecidable whether ***K*** is valid, and thus whether *Kripke's Thesis* is true or false.

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[3] *Step 2* is actually the more complicated: $\neg\exists$*x(x is a grounding-fact & (x => S is true v x => S is false).* The argument below applies equally to this more complicated version, but I use the simpler version in the main text.

Here's the argument that it's undecidable that *S is true* has truth-conditions. There is no specification of a grounding-condition for *S is true*, since the latter is a ground-unspecifiable sentence. But that means that a perfect assertor, searching for what's assertable given perfect information ***I*** about *S*, cannot uncover a specification of a grounding-condition for *S is true*. There just isn't one. So, the perfect assertor cannot affirm ***T***:

> ***T***: There is an *x* such that *x* is a grounding-condition and
> if *x* obtains, *x* => '*S* is true' is true.

So the prefect assertor cannot affirm that *S is true* has truth-conditions. So, *S is true* cannot be said to have truth-conditions.

A very similar argument can be given that it cannot be said that *S is true* lacks truth-conditions. For, by ***TC***, that requires affirming: *There is no y, such that y is a grounding-condition, and if y obtains, y => 'S is true' is true.* For this to be assertable, we have to affirm *'S is true' is true* has truth-conditions, and so, there must be, by ***TC***, a condition, *x* such that x => *'S is true' is true* is true. If there is such a condition, there is a condition, *x* that is grounding, such that, if x obtains, *x* => *S* is true. But *S* is a ground-unspecified sentence. Therefore, that condition cannot be specified. In which case, the perfect assertor, given information ***I***, cannot assert *'S is true' lacks truth-conditions*.

To conclude: it's undecidable that *S is true* has truth-conditions. Therefore, it's undecidable that *step 2* has truth-conditions. If it's undecidable that *step 2* has truth-conditions, then, by *Inference-Rule*, we cannot use it in any logical conclusion. If so, we cannot say that ***K*** is valid, nor that *Kripke's Thesis* is true or false. If so, the proof, ***TM-Refute***, against ***TM*** cannot proceed. We have shown its soundness is undecidable.[4]

We have strong, intuitive reasons, for thinking ***TM*** is correct. There is no threat from *TM-Refute*. So ***TM*** stands.

[4] Note that if we assume that *Kripke's Thesis* can be said to have truth-conditions, we can affirm that *Kripke's Thesis* entails ***TM*** is false, and so we can infer ***TM*** entails that *Kripke's Thesis* is false. Fortunately, the assumption in question, that *Kripke's Thesis* can be said to have truth-conditions, cannot be discharged. So we cannot draw the conclusion that *Kripke's Thesis* is false via this route.

V
Paradox and alethic undecidability

Given ***TM***, we cannot infer that ground-unspecifiable sentences lack truth-makers, or false-makers. Therefore, contra Kripke (1975), we cannot infer that they suffer truth-value gaps. But we cannot infer that they are bivalent either, given ***TM***. Indeed, given ***TC***, it's undecidable that ground-unspecifiable sentences have truth-conditions. So they are alethically undecidable. What then can we conclude about semantic paradoxes?

Paradoxical sentences are ground-unspecifiable sentences. (That's easily verified, given their looping and infinitely descending referential chains.) Kripke's (1975) strengthened liar *L* is ground-unspecifiable:

L : *L* is not true.

By ***TM*** and ***TC*** we must conclude that it's alethically undecidable. We cannot say it is true, false, that is lacks truth-value, or that it's both true and false. Indeed, we cannot say if *L* has any truth-conditions or not.[5] In particular we cannot use the *T*-schema and classical logic to derive a contradiction. That's because the required *T*-sentence is alethically undecidable, because it's undecidable that it's constituent sentences, *L is true* and *L is not true*, have truth-conditions, since they are ground-unspecifiable sentences:

$\boldsymbol{T^L}$: *L* is true iff *L* is not true.

Therefore, we must refrain from asserting $\boldsymbol{T^L}$.

You might object that this means denying the validity of the *T*-schema. It does not. Logical principles concern sentences that *decidably* have truth-conditions—that accords with ***Inference-Rule*** above. After all, logic concerns the laws of thought. So, we should think of the *T*-schema's

[5] This last conclusion is in conflict with the theorists like Goldstein (1999) who argue that Liar-sentences express no proposition. But that conclusion is unstable. If *L* expresses no proposition, then a commentary sentence like *C* affirms a truth-about it. But *C* and *L* are grammatically and referentially identical. So why it is that *L* expresses no proposition, whereas *C* does? I don't think Goldstein has a satisfactory answer. Goldstein thinks *L* cannot express a proposition because supposing *L* expresses a proposition generates a contradiction (assuming propositions are either true or false). But his argument ignores the possibility of indeterminacy: that one cannot say either way, whether it does or does not express a proposition.

validity thus: the *T*-schema is valid if and only if for all sentences *S* whose possessing truth-conditions *is decided* the corresponding *T*-sentence is true. On that understanding the *T*-schema is valid, despite $\boldsymbol{T}^{L}$'s not being assertable.

Incoherence?

Another objection is that the ***TM***-solution is incoherent. In order to affirm (*C*) *One cannot say that L is true*, we are committed to the embedded sentence *L is true* having truth-conditions. But the latter's truth-conditional status is meant to be undecidable. The objection fails. Asserting *C* does not commit us to *L is true* having truth-conditions. (Compare *Step 2*. Given the meanings of '&', '∃*x*' and '¬' we can deduce that the truth-conditions of *step 2* depends on *S is true*'s having truth-conditions—see main text. But there is no comparable argument for *C* and *L is true*.) *C*'s assertability only commits us to *L is true* being meaningful (which it is). The undecidability solution, given ***TC***, does not affirm an identity between a sentence's saying something and its having-conditions (the central hypothesis of truth-conditional semantics). In short, there is an implicit challenge to truth-conditional semantics in this solution.

Revenge

Although standard strengthened liars like *L* won't generate a revenge contradiction for the alethic undecidability approach, other kinds of revenge sentence might appear to threaten it. Consider *R*:

R : One cannot assert that *R* is true.

Does *R* have truth-conditions? Although *R* uses an alethic predicate, *true*, *R* does not describe an alethic condition. It's about our capability of asserting that a sentence is true, not about a sentence being true or false, and so on. So, *R* is not a ground-unspecifiable sentence like *L*. However, when we attempt to ascertain whether what *R* says is the case or not, we enter into a *processing loop*. Suppose that we can say *R* has truth-conditions. If *R* is true, it's because the condition *R** holds:

*R**: *One cannot say that R is true.*

But if the condition R^* obtains, it does so because the denotation of 'R' meets one of three conditions: it's false, it lacks truth-conditions, or it's undecidable that it has truth-conditions. To determine if one of these conditions obtains we must return to R itself, and ask about its truth-conditions. Which brings us back to R^*. That's the processing loop. This means that an ideal assertor given information ***I*** will not be able to assert that R^* obtains or that R^* does not obtain, despite the fact that ***I*** records perfectly non-alethic reality, truth's dependence, and all the facts about R's linguistic meaning and referential properties. That's because the ideal speaker will get caught up in the processing loop. We must conclude then:

M : It's undecidable that R^* obtains or not.

What follows from M? Nothing. There's no revenge, just a higher-order indeterminacy. It's undecidable whether the truth of R is undecidable. We cannot say that it's true that we cannot say that R is true. We cannot say that R follows from M.

Conclusions

There is a lot more to say here about the nature of alethic undecidability, and its implications for how we understanding truth, semantic properties, logic, and semantics. But that's enough for now. What's been shown is that the basic principle of truth-making, ***TM***, implies a new kind of solution to the semantic paradoxes, one quite distinct from that developed by Kripke's (1975) reflections on truth. Whether or not this form of solution can be generalized to other cases cannot be demonstrated here. My hunch is that it can.

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